

PHIL 4310: Advanced Logic
Spring 2026
Homework 7

This homework is due Mon, May 11th.

Part 1: Material Conditional semantics

For each of these sentences, say whether it is a tautology, a contradiction, or neither.

1.1 $A \supset \sim A$

1.2 $(A \supset B) \wedge (A \supset \sim B)$

1.3 $(A \supset B) \vee (A \supset \sim B)$

1.4 $(A \supset B) \wedge (\sim A \supset B)$

1.5 $(A \supset B) \vee (\sim A \supset B)$

1.6: Produce a material conditional sentence (main connective ' \supset ') which is a contradiction. Explain why it is a contradiction. What particular features does this sentence have? (I believe if you produce one you will know what features I mean). Is it possible for a material conditional to be a contradiction without possessing those features? Either give an example or give an argument there can't be one.

1.7: Prove that if $\mathbf{Ac} \models \mathbf{Ap}$ and $\mathbf{Cp} \models \mathbf{Cc}$ then $\mathbf{Ap} \supset \mathbf{Cp} \models \mathbf{Ac} \supset \mathbf{Cc}$

(here \mathbf{Ac} is the antecedent of the conclusion, \mathbf{Cp} is the consequent of the premise, etc.)

Part 2: Conditional probabilities

For each of these problems, assume that $0 < P(A) < 1$.

2.1 Show that $P(A|A) = 1$ and $P(A|\sim A) = 0$.

2.2 Consider the argument C therefore $A \supset C$. This is deductively valid. Now show that if $P(C) = 1$, then $P(C|A) = 1$.

2.3 Now show that this argument is not probabilistically valid. Do this by giving a stochastic truth table where $P(C) = .9$ but $P(C|A) = 0$. (Recall that a probabilistically valid argument is one where the uncertainty of the conclusion cannot exceed the sum of the uncertainty of the premises when the probabilities of any conditionals are replaced by the corresponding conditional probabilities).

Part 3: Stalnaker's Conditional

For each of these arguments, if they are invalid on Stalnaker's semantics for the

conditional, give a countermodel. If they are valid, give an argument (informal in English/logic) that they are valid.

3.1 $A \rightarrow B, A \rightarrow C$ therefore $A \rightarrow (B \wedge C)$

3.2 $A \rightarrow C$ therefore $(A \wedge B) \rightarrow C$ [antecedent strengthening]

3.3 $A \leftrightarrow B, A \rightarrow C$ therefore $B \rightarrow C$

3.4 $(A \vee B) \rightarrow C$ therefore $(A \rightarrow C) \wedge (B \rightarrow C)$

3.5 $(A \vee B) \rightarrow C$ therefore $(A \rightarrow C) \vee (B \rightarrow C)$

NOTE: By the way, every one of these arguments is valid for the material conditional.

Part 4: Counterfactuals

One key difference between Lewis's and Stalnaker's semantics for counterfactuals is that Stalnaker assumes that there is a unique 'closest' A-world (if there is one at all). For many inferences, this does not make a difference. But it does for some. For the problem below, explain why it is valid in Stalnaker's semantics but not in Lewis's.

4.1 $A \Box \rightarrow (B \vee C), \sim(A \Box \rightarrow B)$ therefore $A \Box \rightarrow C$

NOTE: There is a quantified version of this as well. Here is close parallel:

$A \Box \rightarrow \exists xFx$ therefore $\exists x(A \Box \rightarrow Fx)$. This is valid for Stalnaker, not for Lewis.

Here is an example discussed in the literature: Jimmy Carter never appointed anyone to the supreme court as there were no vacancies during his term. However, it is widely believed that had a vacancy occurred, he would have appointed a woman. In fact, the next vacancy was in 1981, and Reagan appointed Sandra Day O'Connor who was the first woman on the court. So it might be true that $\text{Vac} \Box \rightarrow \exists x(Wx \wedge CAx)$. But does that imply that there is a specific woman who is such that had there been a vacancy, she would have been appointed?